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The book is composed by abstracts on problems of classical mechanics, theory of stability and bifurcations, regular and chaotic dynamics. Oscillations of mechanical systems, dynamics of a rigid and flexible bodies, three and N bodies problems, periodic and almost periodic orbits, natural and artificial celestial bodies dynamics including dynamics of orbital systems, computer methods of simulation in dynamics including engineering applications are under consideration. Special attention paid to the problems of modeling and simulation of the multibody systems dynamics.

Canonical Formalism in Problem for Motion of Incompressible Liquid Gravitational Ellipsoid

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Consider an affine deformation of the ellipsoid. A state of the ellipsoid at given time characterizes a matrix A

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}, \tag{8}$$

which transforms "primary body" into the ellipsoid:

$$\tilde{\mathbf{r}}(t) = \mathbf{A}(t)\tilde{\mathbf{R}}, \tag{9}$$

where $\tilde{\mathbf{R}}$ is the fixed primary vector, $\tilde{\mathbf{r}}$ is the vector, connected with moving fluid particle. The primary body is the sphere of radius $R = R_0$

$$\tilde{\mathbf{R}}\mathbf{E}\tilde{\mathbf{R}} = R_0^2. \tag{10}$$

From (9) it follows that $\tilde{\mathbf{R}} = \mathbf{A}^{-1}\tilde{\mathbf{r}}$. Substitute this relation into (10):

$$\tilde{\mathbf{R}}\mathbf{M}\tilde{\mathbf{R}} = R_0^2,$$

where $\mathbf{M} = (\mathbf{A}^*\mathbf{A})^{-1}$, so that in the principal axes one finds

$$\mathbf{M} = \begin{pmatrix} a_1^{-2} & 0 & 0 \\ 0 & a_2^{-2} & 0 \\ 0 & 0 & a_3^{-2} \end{pmatrix}; \quad \mathbf{M}^{-1} = \begin{pmatrix} a_1^2 & 0 & 0 \\ 0 & a_2^2 & 0 \\ 0 & 0 & a_3^2 \end{pmatrix}.$$

The gravitational energy of the ellipsoid reduces to the form

$$W = -\frac{3G\mu^2}{10}\int_0^\infty \left[\det\left(\mathbf{M}^{-1} + s\mathbf{E}\right)\right]^{-1/2} ds,$$

where μ is the total mass, and $\mathbf{E}$ is the unit matrix. The kinetic energy reads

$$T = \frac{4\pi\rho}{30}\mathrm{Sp}\dot{\mathbf{A}}^*\dot{\mathbf{A}} = \frac{\mu}{10}\mathrm{Sp}\dot{\mathbf{A}}^*\dot{\mathbf{A}}.$$

Besides, there is the compression energy

$$D = h\left(\det\mathbf{A} - \kappa\right)^2,$$

where h is a certain constant, κ is the "standard" volume growth while the conversion of the sphere into the ellipsoid. Then, we make the Hamiltonian

$$H = T + W + D = \tfrac{\mu}{10}\mathrm{Sp}\dot{\mathbf{A}}^*\dot{\mathbf{A}} - \\ -\tfrac{3}{10}\mu^2 G \int\limits_0^\infty \left[\det\left(\mathbf{M}^{-1} + s\mathbf{E}\right)\right]^{-1/2} ds + h\left(\det\mathbf{A} - \kappa\right)^2.$$

The generalized coordinates in this problem a_{ik} are the elements of the matrix $\mathbf{A}$ from (8); the generalized momenta b_{ik} are the elements of the matrix $\frac{1}{5}\mu\dot{\mathbf{A}}$. It is possible to show, that

$$\frac{da_{ik}}{dt} = \frac{\partial H}{\partial b_{ik}} = \dot{a}_{ik};$$

$$\frac{db_{ik}}{dt} = -\frac{\partial H}{\partial a_{ik}} = \frac{1}{5}\mu\ddot{a}_{ik},$$

where

$$\ddot{a}_{ik} = -\tfrac{10h}{\mu}\left(\det\mathbf{A} - \kappa\right)\left(\mathbf{A}^*\right)^{-1} + \\ +\tfrac{3G\mu}{2}\int\limits_0^\infty \left\{\det\left(\mathbf{M}^{-1} + s\mathbf{E}\right)\right\}^{-3/2}\left\{\left(\mathbf{M}^{-1} + s\mathbf{E}\right)\mathbf{A}\right\}_{ik} ds.$$

References

1 Kondratyev, B. P., The Potential Theory and Equilibrium Figures. Moscow–Izhevsk, RCD, 2003.

The Hertz Contact Problem and Its Application to the Vehicle Dynamics Computer Simulation

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A method of computer implementation of an elastic contact model for rigid bodies in frame of the Hertz contact problem [1] is considered. An algorithm to transform the outer surfaces geometric properties to the local contact coordinates system is analyzed in details [2]. This plays an important role when applying the object-oriented approach to simulate the process of contacting.